Electrical Measurements

Code: EPM1202

Lecture: 4 Tutorial: 2 Total: 6

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Basic definitions

Instrument efficiency

The ratio between the full scale and the consumed power during reading full scale

For the voltmeter:

$$\eta = \frac{V_{fs}}{P_{fs}} = \frac{V_{fs}}{\mathrm{V}_{\mathrm{fs}}^{2}/\mathrm{R}_{\mathrm{m}}} = \frac{\mathrm{R}_{\mathrm{m}}}{V_{fs}}$$

Where: R_m is the meter resistance
V_{fs} is the full-scale reading
P_{fs} is the consumed power at full scale

The voltmeter efficiency increases with the increase of its internal resistance

Basic definitions

Instrument efficiency

For an ammeters, the full-scale reading is assumed to be I_{fs} and the internal resistance is R_m

$$\eta = \frac{I_{fs}}{P_{fs}} = \frac{I_{fs}}{\mathrm{I}_{\mathrm{fs}}^2\,\mathrm{R}_\mathrm{m}} = \frac{1}{I_{fs}\,\mathrm{R}_\mathrm{m}} \qquad \mathrm{volt}^{-1}$$

The ammeter efficiency increases with the decrease of its internal resistance

Example: A measuring instrument has a full scale of 500V and an error of ± 1.0%. Calculate the instrument accuracy and percent accuracy. What is the percentage accuracy if the instrument is used to read: 250V and 100V

Solution: $\delta_r = 0.01 = \delta_o / A_t = \delta_o / 500$

$\delta_o = \pm 0.01 * 500 = \pm 5V$

$A = 1 - \delta_r = 1 - 0.01 = 0.99$

The percent accuracy:

a = 100 – percent relative static error = 100-1.0 = 99%

Solution (cont.):

$\delta_o = \pm\, 0.01 * 500 = \pm 5V$

$A = 1 - \delta_r = 1 - 0.01 = 0.99$

The percent accuracy:

a = 100 – percent relative static error = 100-1.0 = 99%

At 250V reading

Reading $= A_m = A_t + \delta_o = 250 \pm 5 = 245$ V or 255 V

$\delta_r = \delta_o / A_t = 5 / 250 * 100 = 2\%$

$a = 100 \pm 2 = 98\%$ or 102 %

At 100V reading

Reading $= A_m = A_t + \delta_o = 100 \pm 5 = 95$ V 0r 105 V

$\delta_r = \delta_o / A_t = 5 / 100 * 100 = 5\%$

$a = 100 \pm 5 = 95\%$ or 105 %

Example:

In a certain experiment, the measured resistance was 3456 Ω. The used resistor box contains the following resistors: 10 of 1000 Ω, ±0.1% 10 of 100 Ω, ±0.2%

10 of 10 Ω, ±0.4% 10 of 1 Ω, ±0.6%

Calculate the percentage uncertainty in the reading

Solution:

The error in measurement is:

3000*(0.1/100)+400*(0.2/100)+50*(0.4/100)+6*(0.6/100)

= 3 +0.8 + 0.2 + 0.036 =4.036 Ω

The percentage uncertainty = ±4.036 / 3456 * 100

= 0.1168%

Example:

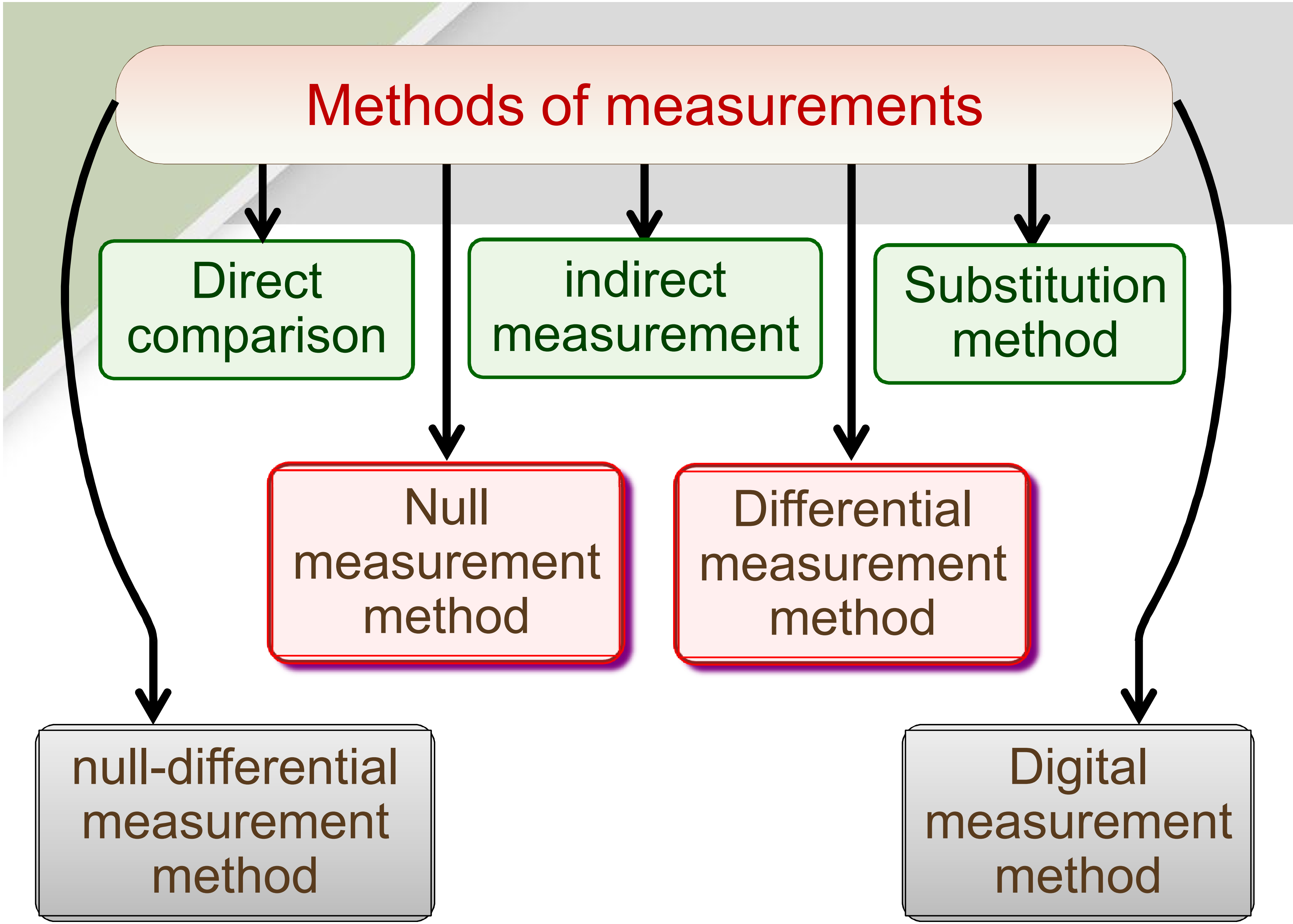
During a measurement of a current, a change of 2 A causes a deflection of 3mm on the ammeter scale. Calculate the Ammeter sensitivity.

Solution:

The sensitivity is given as:

S = magnitude of input / magnitude of output

$$= 2 / 3 = 0.6667 \text{ A /mm}$$


Methods of measurements
Direct comparison
indirect measurement
Substitution method
Null measurement method
Differential measurement method
null-differential measurement method
Digital measurement method

Methods of measurements

Direct comparison

High accuracy with low uncertainty

Requires trained operator and relatively large time to carry out the measurement

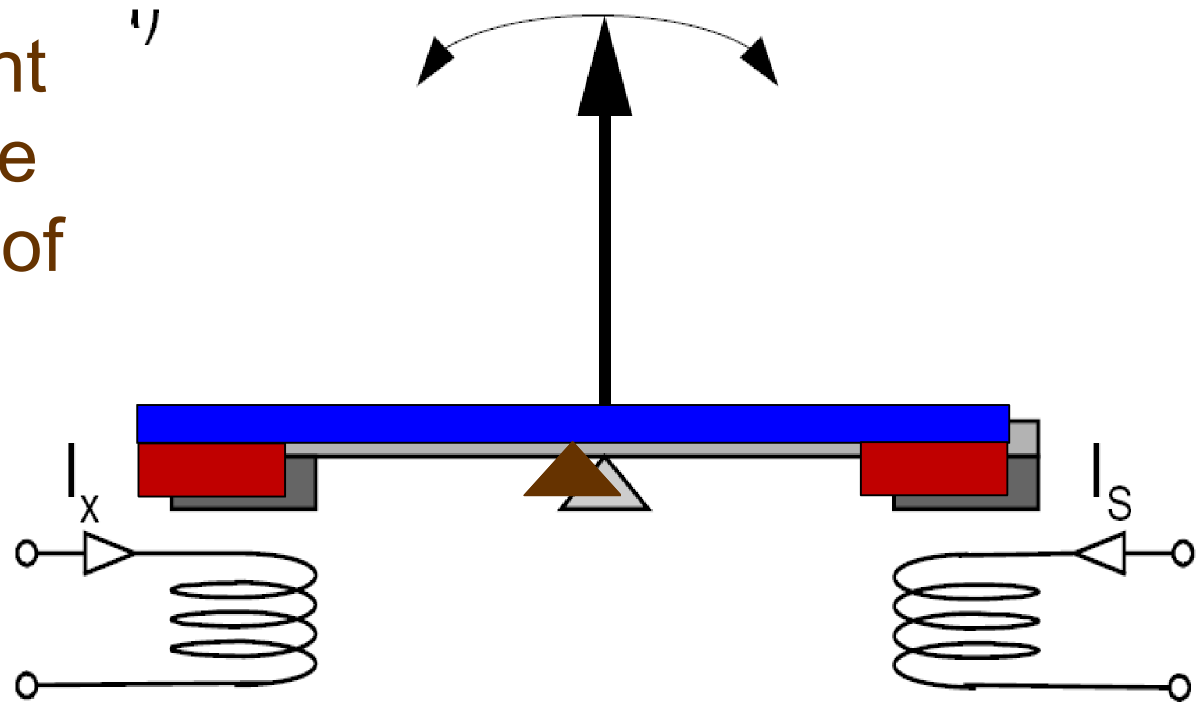
Unknown current can be measured by comparing its magnetic effect with a standard current

Methods of measurements

Direct comparison

The unknown current "I_x" passes through one coil of the electromagnet causing an attraction on one arm of the balance

The standard current "I_s" flows through the coil on second arm of the balance causing another attractive force

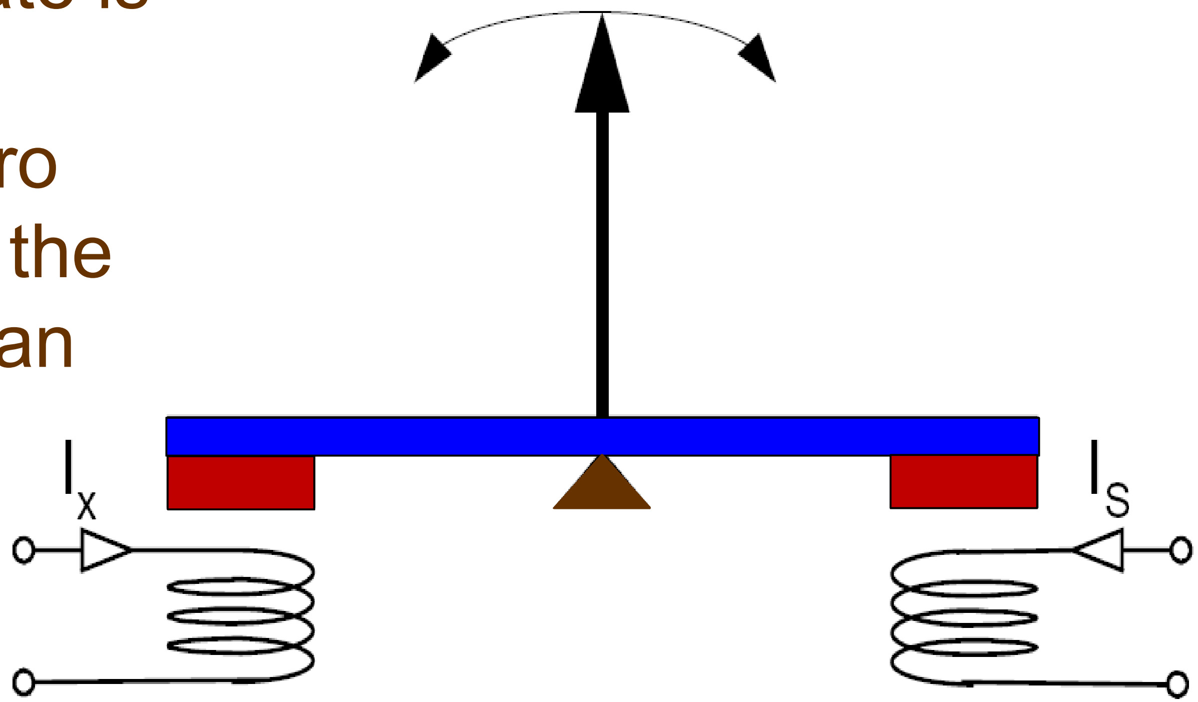


Methods of measurements

Direct comparison

Changing the value of the standard current can cause a balance in the weight

The equilibrium state is reached when the pointer is at the zero position, and thus, the unknown current can be measured



Methods of measurements

Indirect measurement

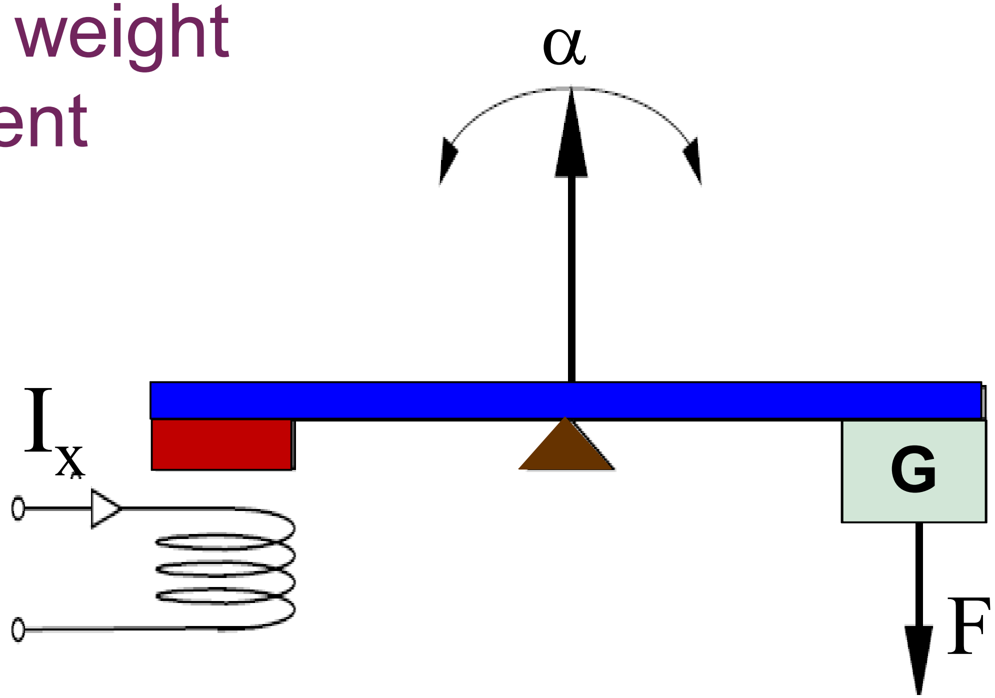
The current can be measured using a weight

The pointer movement depends on arm attraction by the force depending on the measured current “Ix”

The force of gravity “F” of weight “G” balances this movement

No standard current is used in measurement

This method is employed in indicating instruments



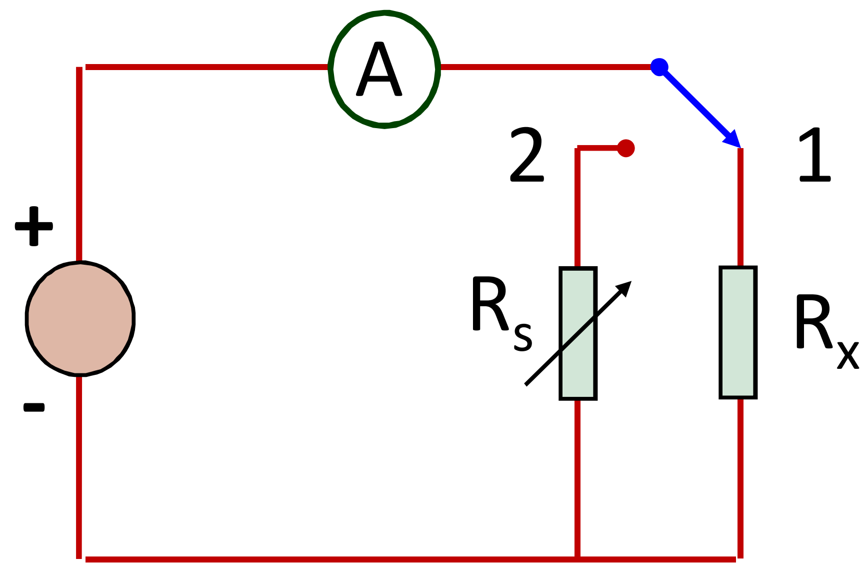
Methods of measurements

Substitution method

It is used to measure the unknown value keeping the same measured value

The current through the unknown resistance R_x is measured by the ammeter with switch in position 1

The switch is then changed to position 2 and the standard resistance R_s is modified until the reading is the same like the first one



Methods of measurements

Null measurement method

A simple, accurate and widely used method

The instrument reading is adjusted to read zero current only and the balance state is indicated by a pointer or electrically by a zero reading

Calculations are required to obtain the unknown value

The balancing process is indicated using a potentiometer or a bridge

The calibration of the meter is unnecessary

A sensitive milliammeter or microammeter with zero-centre position, called a **galvanometer**, can be used

Methods of measurements

Differential measurement “analogue” method

No balance state is required since the deflection of the pointer or the output reading is used as the measure of the value

The application of this method is when continuously monitoring of the value is required

Generally, electromechanical instruments depend on the analogue method

Methods of measurements

Null-differential measurement method

It is a combined measurement method

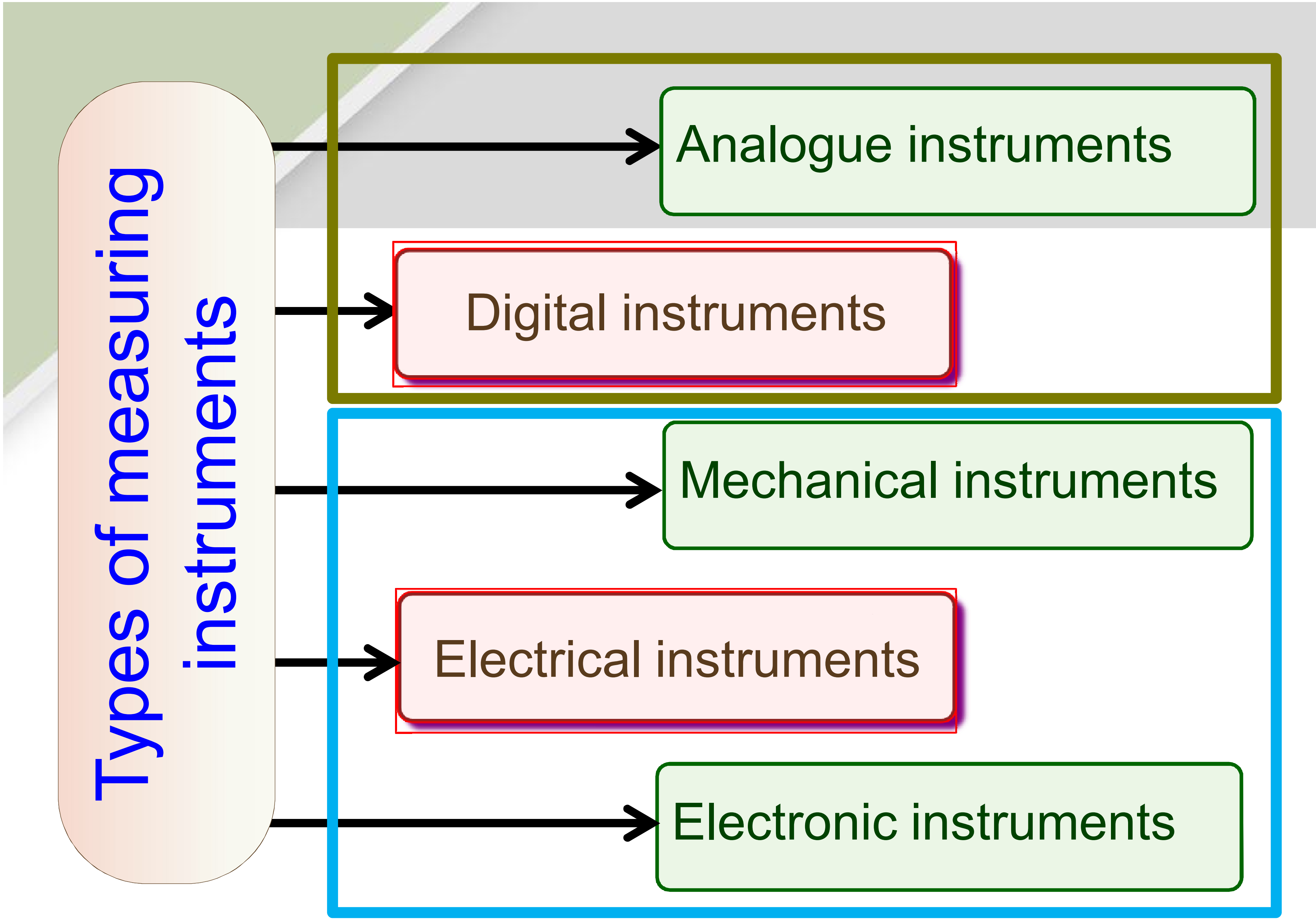
A roughly balance is achieved and a pointer deflection occurs due to the difference between equilibrium and actual states

Using the null-differential method, an improvement of the sensitivity of the measurement can be achieved since the movement of the pointer can be realized by the smaller currents

Methods of measurements

Digital measurement method

It depends on sampling the signal and indicates the reading using discrete numbers


Types of measuring instruments
Analogue instruments
Digital instruments
Mechanical instruments
Electrical instruments
Electronic instruments

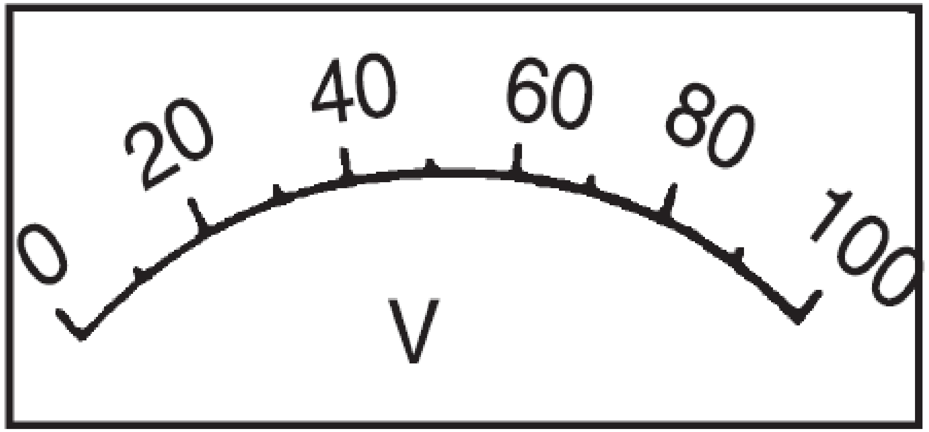
Types of measuring instruments

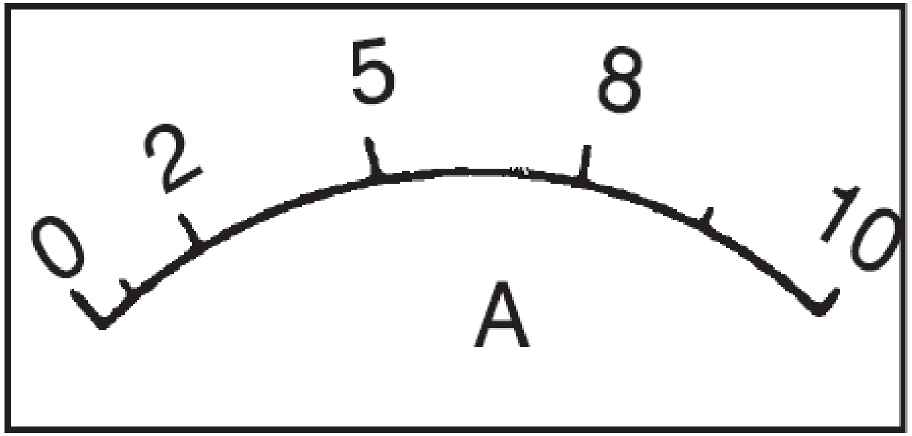
Analogue instruments

They have two types of scales: linear and non-linear.

The divisions or graduations In linear scale instruments are equally spaced

The scale in non-linear scale instruments is overcrowded at the beginning and the graduations are uneven throughout the range





Types of measuring instruments

Digital instruments

They can handle a much wider range of frequency (from d.c. up to MHz)

The accuracy and resolution is higher and automatic range adjustment is possible

Digital instruments provide digital display and can be used for ac and dc measurements

They can measure voltage, current, resistance and other variables at the same time

Types of measuring instruments

Mechanical instruments

These instruments depend on mechanical effects and movements in their operation

They are characterized by their heavy weight, large size and high prices

Due to the normal inertia of mechanical parts, the response of these instruments is slow

They are not suitable for transient and dynamic applications

They can be used to measure steady and stable operation

Types of measuring instruments

Electrical instruments

They comprise electrical circuits and depend on the electromagnetic effects on their operation

A magnetic field is produced using either a permanent magnet or a coil

The reading depends on the interaction between the magnetic and the electric circuit

The response is faster than that of the mechanical instrument with smaller size and weight

There is higher possibility for faults and problems inside the instrument

Types of measuring instruments

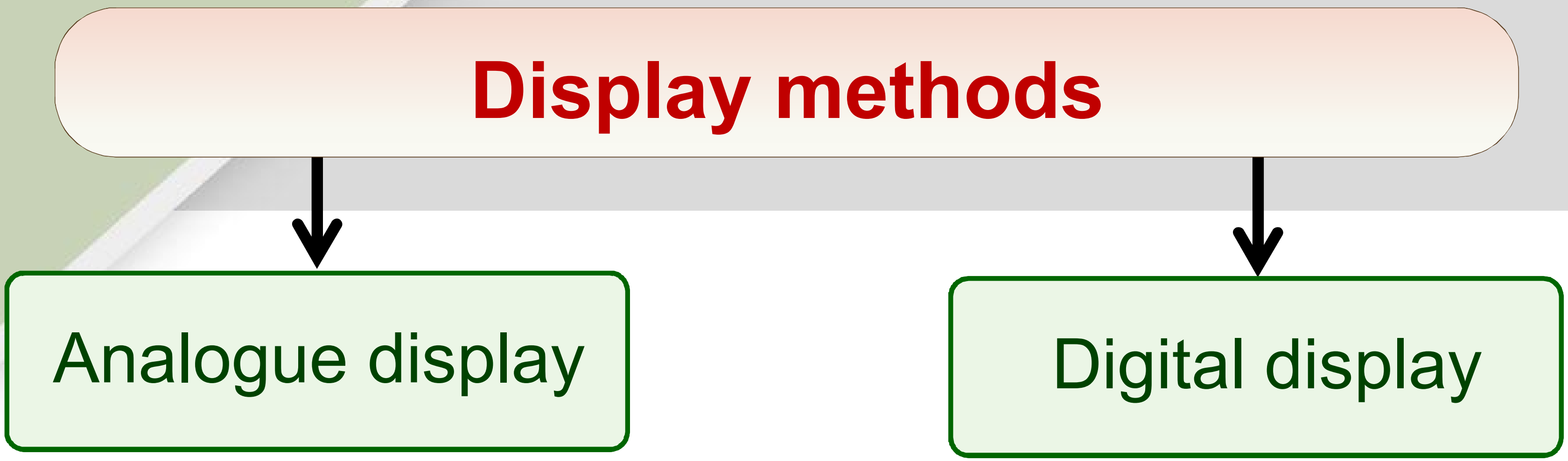
Electronic instruments

Electronic instruments comprise electronic devices and circuits on their operation

The response is very fast due to the use of electronic switches

They can be used with continuously varied measurements

The measured values depend on the rating of the electronic devices such as diodes and transistors



The measured value can be displayed by different methods

The first method is the “analogue display”, which depends on either pointer or graphical display

The second method is the “digital display”, which depends on numeric display

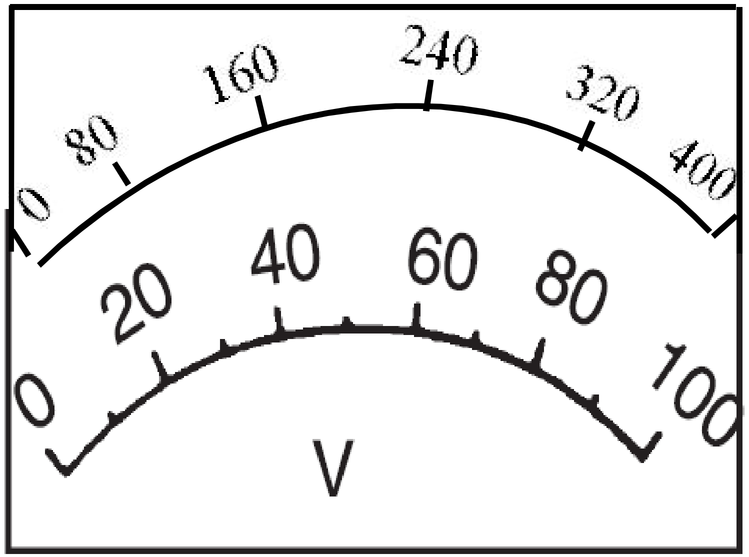
Display methods

Analogue display

Analogue display can be pointer or graphical display

Pointer devices have the problem of "***parallax***", if the operator looks to the pointer with an angle

A mirror is used to help the operator to look with a vertical angle



Sometimes, multi-scale pointer instruments are used and the scales are placed in different levels

Display methods

Analogue display

There is an effective range, where the measurement can be carried out with an acceptable accuracy

The graphical display gives more details for the measured variable since it gives the magnitude in addition to the type of variation

For example, the variation of a variable with time or other variables can be given directly

Display methods

Digital display

The display in this case is achieved in a numerical manner

This gives more accurate and definite reading

It is required in this case to have a digital to analogue and analogue to digital converters

DYNAMIC PERFORMANCE OF ANALOGUE INSTRUMENTS

Dynamic performance of analogue instruments

For analog instruments, the dynamics of the pointer movement has a special importance in evaluating the performance of the instrument

Therefore, it is important to investigate this dynamic regarding the factors affecting this dynamic

Dynamic response

The pointer can not reach its steady state position immediately due to its <u>mechanical nature</u>

A transient period is required until the pointer take up its final steady state position

The steady-state position is an equilibrium state between two torques, where the deflection is caused by the interaction of two fields

The 1st field is due to current flow in the instrument coil

The 2nd field is obtained by a permanent magnet, ferromagnetic vanes, or magnetic field produced by a current flowing in another coil

Dynamic response

This interaction causes a deflecting torque given by:

$$T = K\,f(i) \qquad \text{N.m}$$

Where, T is the deflecting torque, K is a constant and "i" is the current

The deflecting torque is a function of the flowing current

The function depends on the instrument type and the way, by which the torque is produced

Dynamic response

After applying a signal to the instrument, the pointer starts to move towards the steady state value

This movement during the transient period can take different characteristics depending on the instrument

The equation of motion has a dynamic nature and the equation describing the steady state equilibrium is an equality equation

At steady state, the deflecting torque equals the sum of three toques: the inertia torque, the damping torque and the control torque

Dynamic response

$$T = T_i + T_D + T_C$$

Where:

T is the deflecting torque

T_i is the inertia torque

T_D is the damping torque

T_C is the control torque

Dynamic response

Inertia Torque

The moving parts of instrument have a mass and the movement depends on the inertia of this mass "J"

The inertia produces an inertia torque that counteracts the pointer movement during the transient period only, while it will be zero at steady-state conditions

The inertia torque depends on angular acceleration of the pointer but it opposes its direction of motion

Thus, the pointer cannot reach its final position immediately

Dynamic response

Inertia Torque

The dynamic description of the inertia torque can be given using a second order differential equation

$$T_i = J \frac{d^2\theta}{d\,t^2}$$

Where: “J” is the inertia, “θ” is the deflecting angle of the pointer and “t” is the time

Dynamic response

Control Torque

Without controlling "restoring" torque, the deflecting torque will cause a continuous pointer movement

Thus, the pointer would swing over to the maximum deflected position regardless of the measured current

Controlling torque "T_C" opposes the deflecting torque and increases with the deflection of moving system

It restores the pointer back to zero reading when the input signal is removed

Currents of different magnitudes produce deflections of the moving system in proportion to their size

Dynamic response

Control Torque

The pointer comes to rest at the steady state at a position where the two opposing torques are equal

$$\boxed{T = T_c} \Rightarrow K\,f(i) = T_C$$

Controlling "restoring or balancing" torque

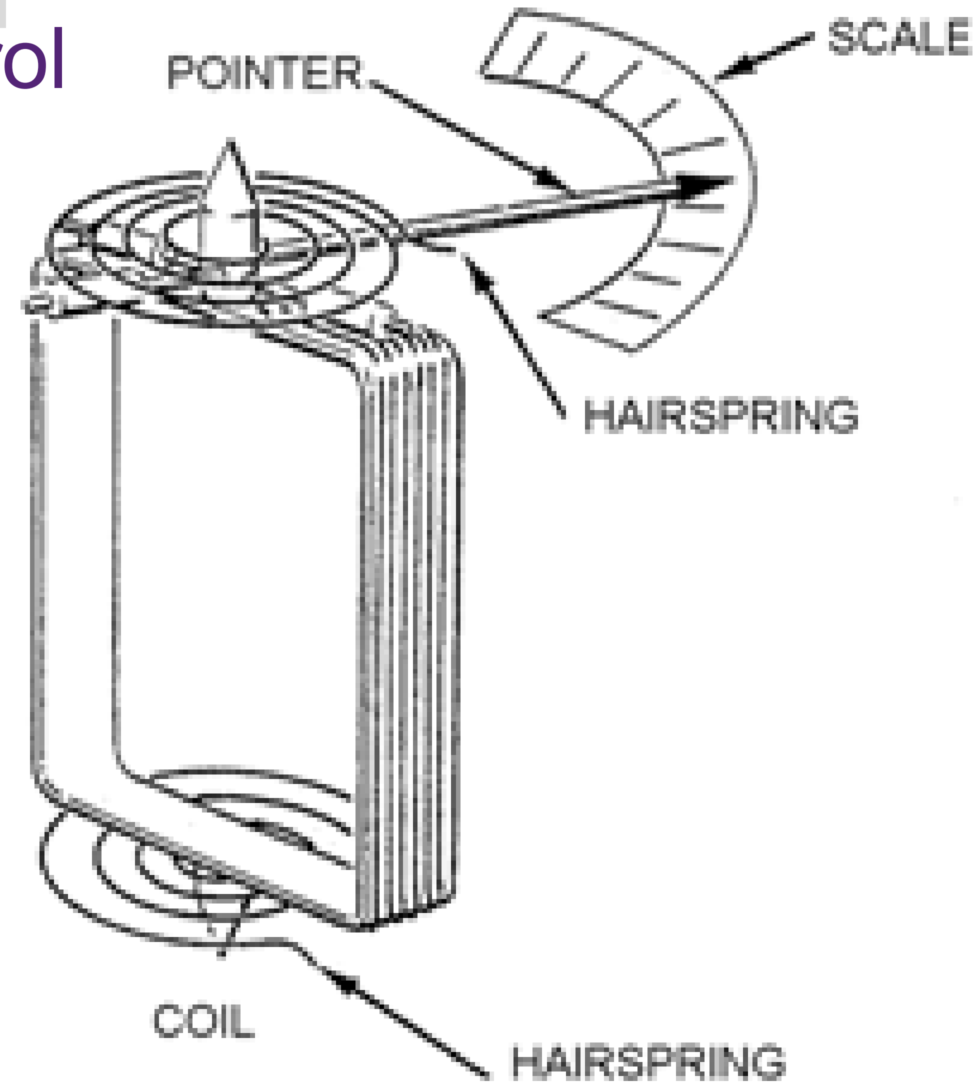
- spring control
- gravity control

Dynamic response

Control Torque

Spring Control

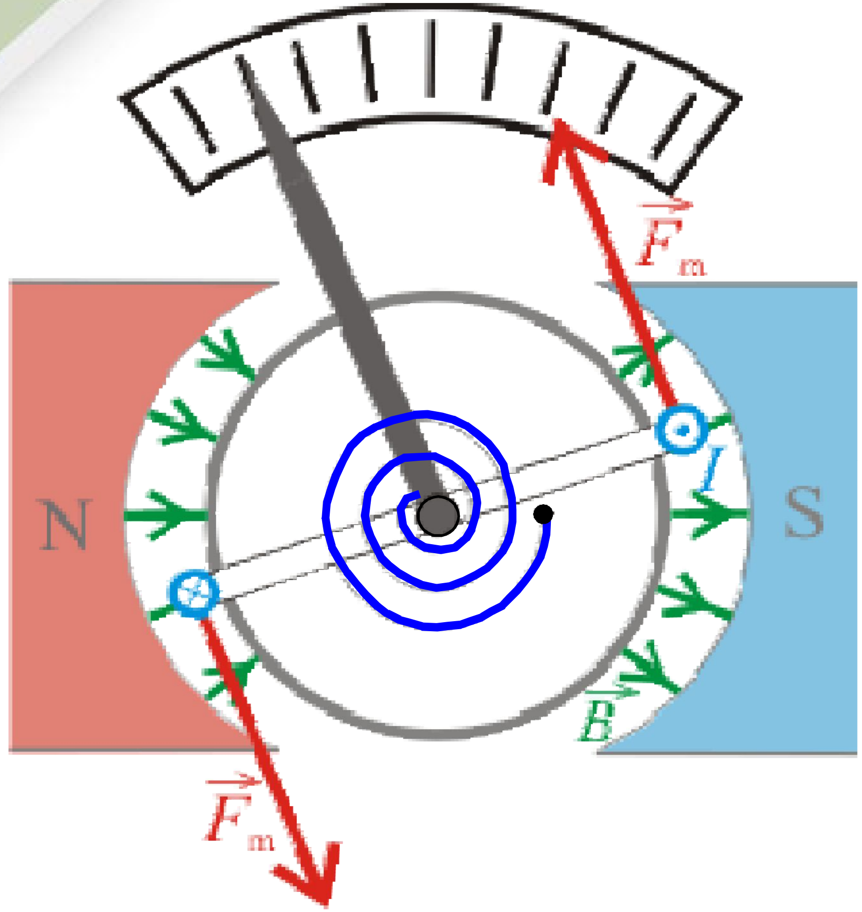
A twisting hairspring, usually of phosphor bronze, is attached to the moving part of the instrument and twists in opposite direction to deflecting torque

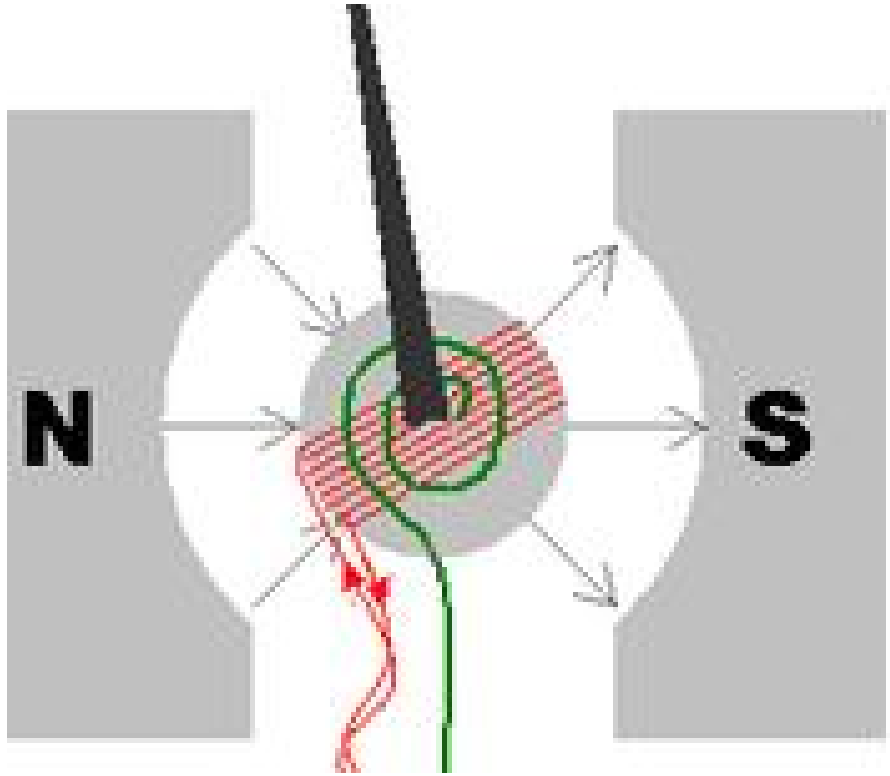


Dynamic response

Control Torque

Spring Control





Dynamic response

Control Torque

Spring Control

With the deflection of the pointer, the spring is twisted in the opposite direction

The spring twist produces restoring torque, which is proportional to the deflection angle

In permanent-magnet moving-coil instruments, the deflecting torque is proportional to the flowing current

$$T \propto I$$

For spring control: $T_c \propto \theta$

Dynamic response

Control Torque

Spring Control

$T \,\alpha\, I$ $\qquad\qquad$ $T_c \,\alpha\, \theta = C\,\theta$

$T_c = T \quad \rightarrow \quad C\,\theta \;\alpha\; I$

$\theta \,\alpha\, I$

The last relation indicates that the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range

Dynamic response

Control Torque

Spring Control

Springs are made of materials with the following characteristics:

They are non-magnetic

They are not subjected to much fatigue

They have low specific resistance

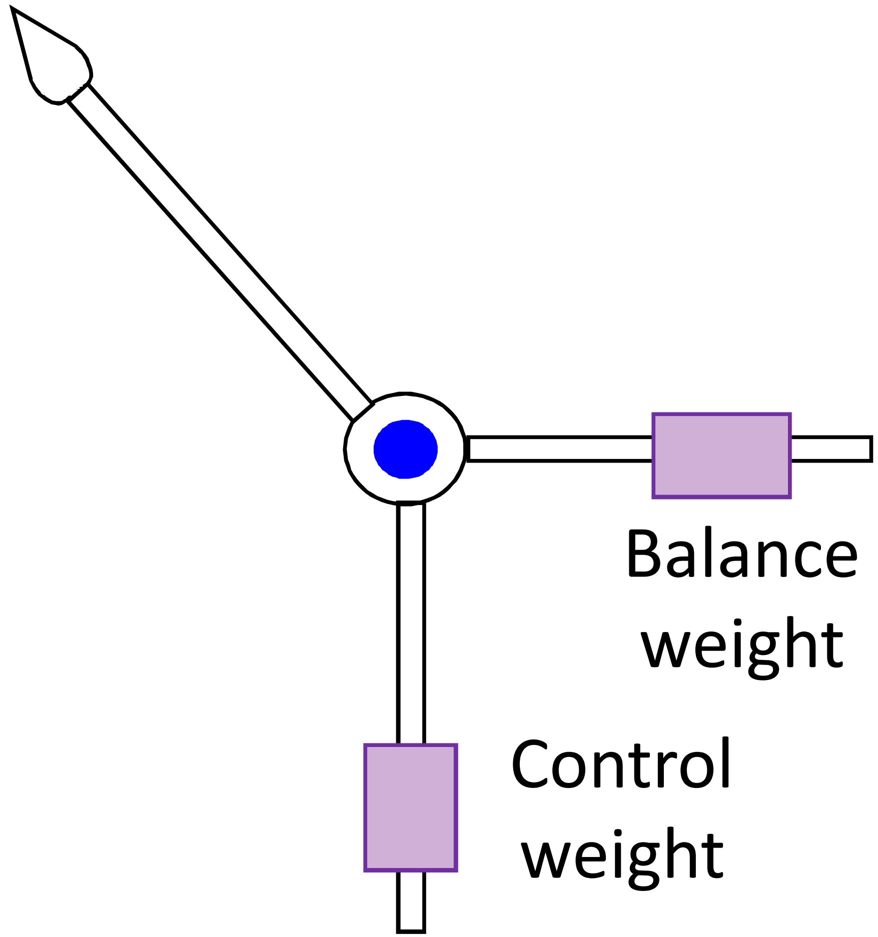
They have low temperature-resistance coefficient

Dynamic response

Control Torque

Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system “pointer terminal” such that the two torques are in opposite directions



Dynamic response

Control Torque

Gravity Control

The controlling or restoring torque is proportional to the sine of the angle of deflection

$$T_c \propto \sin \theta$$

$$T_c = C \sin \theta$$

$$I \propto \sin \theta$$

Dynamic response

Control Torque

Gravity Control

$$I \alpha \sin \theta$$

The current in these instruments are proportional to the sine of the angle not the angle itself

Gravity-controlled instruments have non-uniform scales with crowded scales at its lower end

Dynamic response

Control Torque

Gravity Control

Disadvantages:

They have crowded scale

They have to be kept vertical

Advantages

They are cheap

They are unaffected by temperature

They are not subjected to fatigue or deterioration with time

Dynamic response

Example

For a given ammeter, the deflecting torque is in proportional with the square of the current. A current of 2 A produces a deflection angle of 90°. What is the required current to produce a deflection angle of 45°? Assume that the instrument has:

i) Spring control

ii) Gravity control

Dynamic response

Solution

The deflecting torque is in proportional with the square of the current

$T \propto I^2 \quad \rightarrow T = K I^2$

i) for Spring control

$T \propto \theta$

$$\boxed{\frac{T_1}{T_2} = \frac{I_1^2}{I_2^2} = \frac{\theta_1}{\theta_2}} \Rightarrow \boxed{\frac{2^2}{I_2^2} = \frac{90}{45}} \Rightarrow I_2 = 1.4142 \text{ A}$$

Dynamic response

Solution (cont.)

ii) for gravity control

$$T = C \sin(\theta)$$

$$\boxed{\frac{T_1}{T_2} = \frac{I_1^2}{I_2^2} = \frac{\sin(\theta_1)}{\sin(\theta_2)}} \Rightarrow \boxed{\frac{2^2}{I_2^2} = \frac{\sin(90)}{\sin(45)}}$$

$$I_2 = 1.682 \text{ A}$$

Dynamic response

Damping Torque

The damping torque opposes the moving parts of the instrument when it is in a moving state

It acts to stabilize the motion and to bring the pointer to rest quickly by preventing the pointer oscillations around its final position due to inertia effect

High damping results in high time till equilibrium, while low damping causes high oscillations

The damping degree is adjusted to enable the pointer to rise quickly to its deflected position without over shooting

Dynamic response

Damping Torque

$$T_d = D\frac{d\theta}{dt}$$

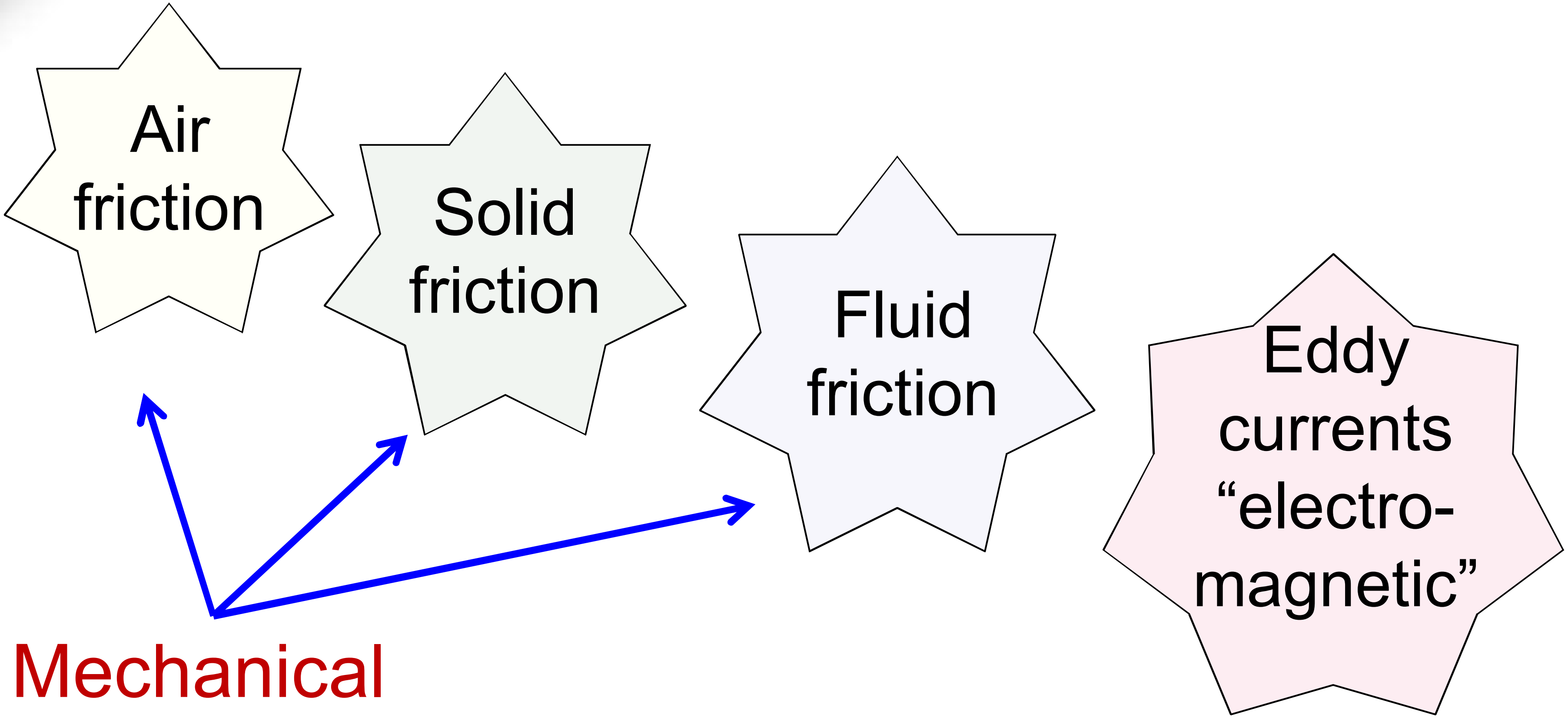
T_d is the damping torque

D is the damping constant

The damping constant depends on the applied damping mechanism

Dynamic response

The damping torque can be produced by



Dynamic response

Damping Torque

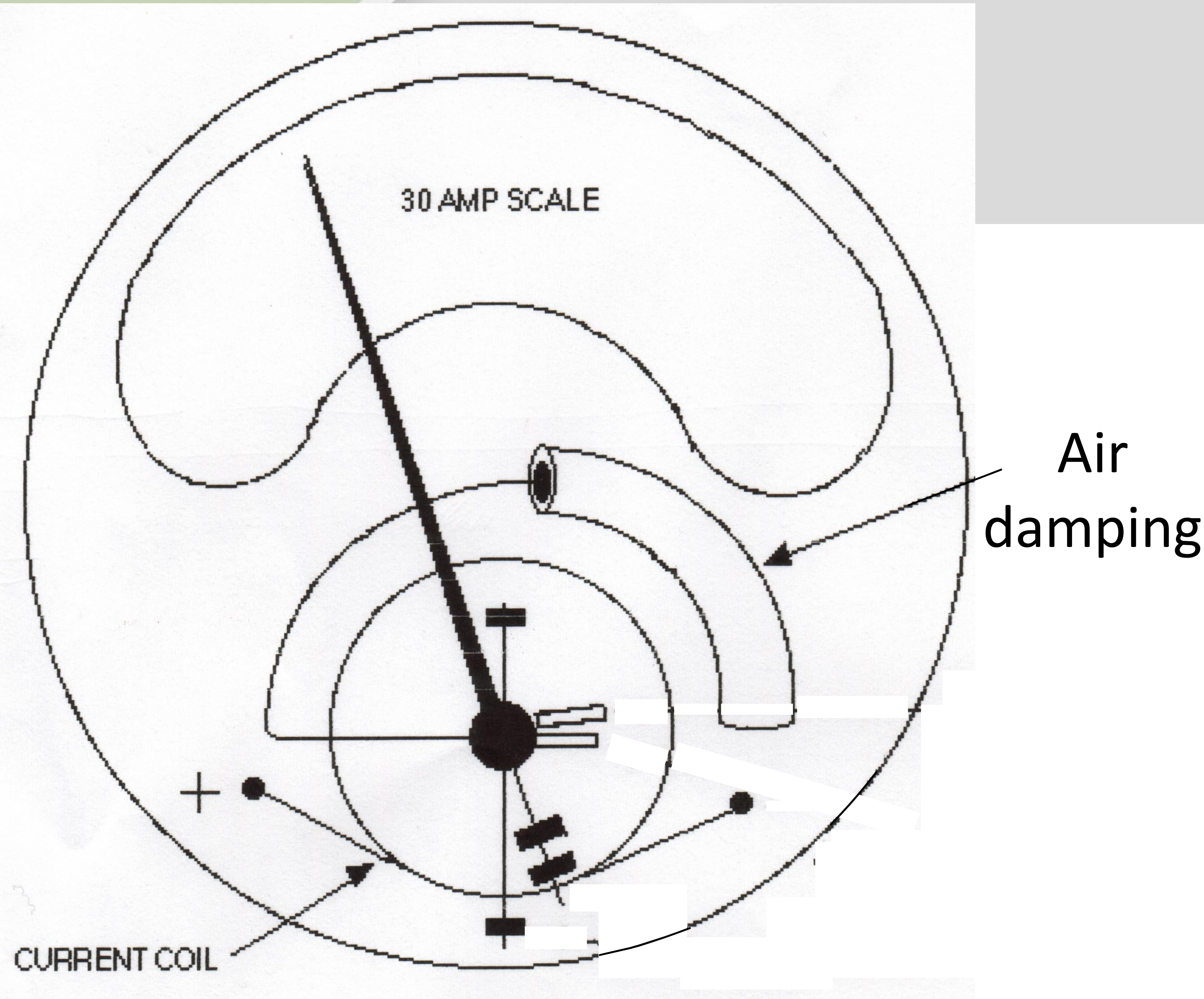
Mechanical damping

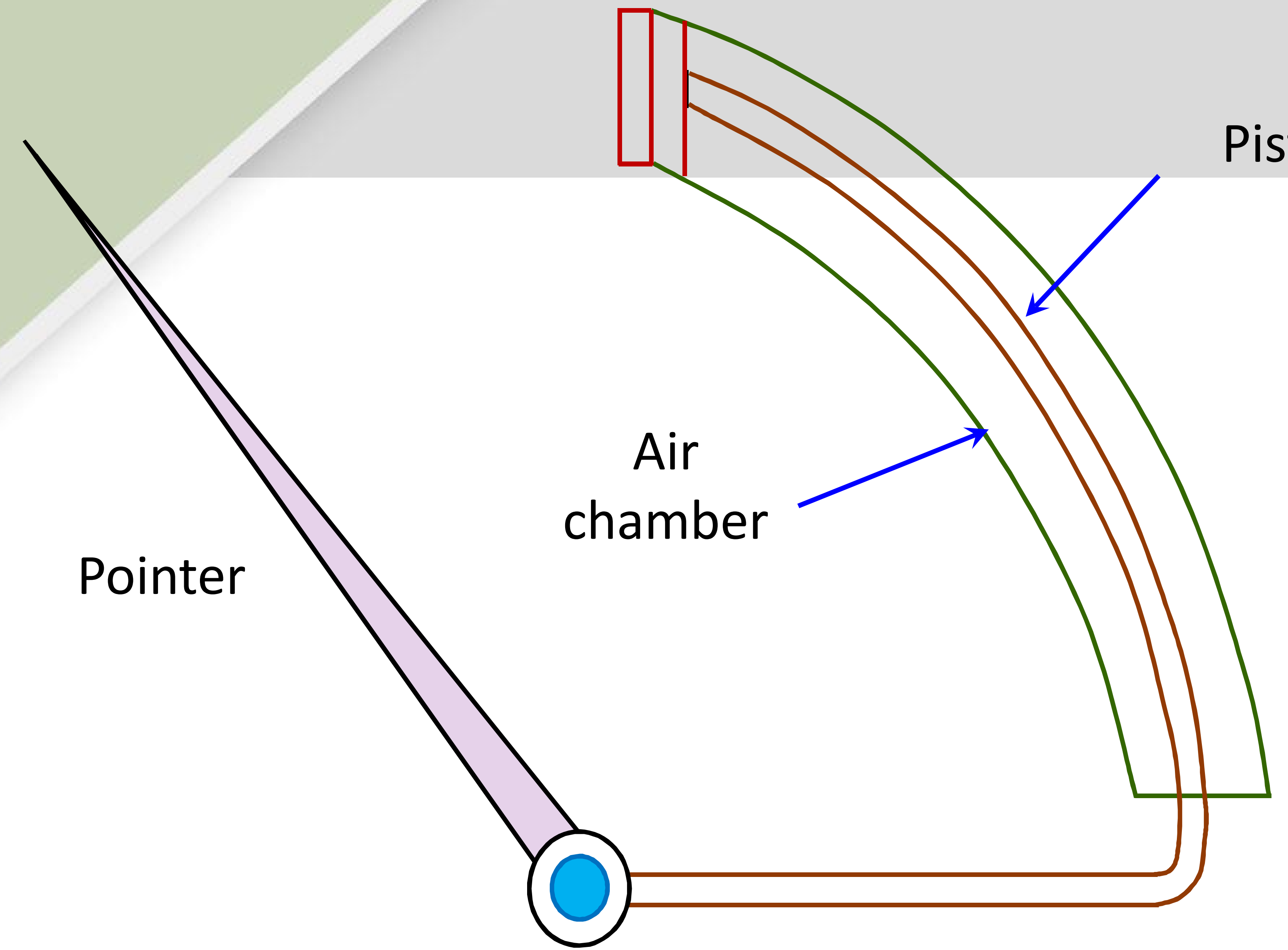
Air damping

Achieved though the motion of an aluminium vane in air chamber depending on the mechanical movement and independently of the coil current

The aluminium piston attached to the moving system moves with a very small clearance in a fixed air chamber closed at one end

The chamber cross-section is circular or rectangular


30 AMP SCALE
Air damping
+
CURRENT COIL


Piston
Air
chamber
Pointer

Dynamic response

Damping Torque

Mechanical damping

Air damping

The compression and suction actions of the piston on air in the chamber affect the oscillations damping

Air damping is not effective in many situations

During the movement of the pointer, the air contained in the air chamber resists the movement and hence causes a damping

Dynamic response

Damping Torque

Mechanical damping

Liquid damping

The motion of an aluminium vane is in viscous fluid

It is independent of the current flowing through the coil

The damping is more effective due to the higher viscosity of oil

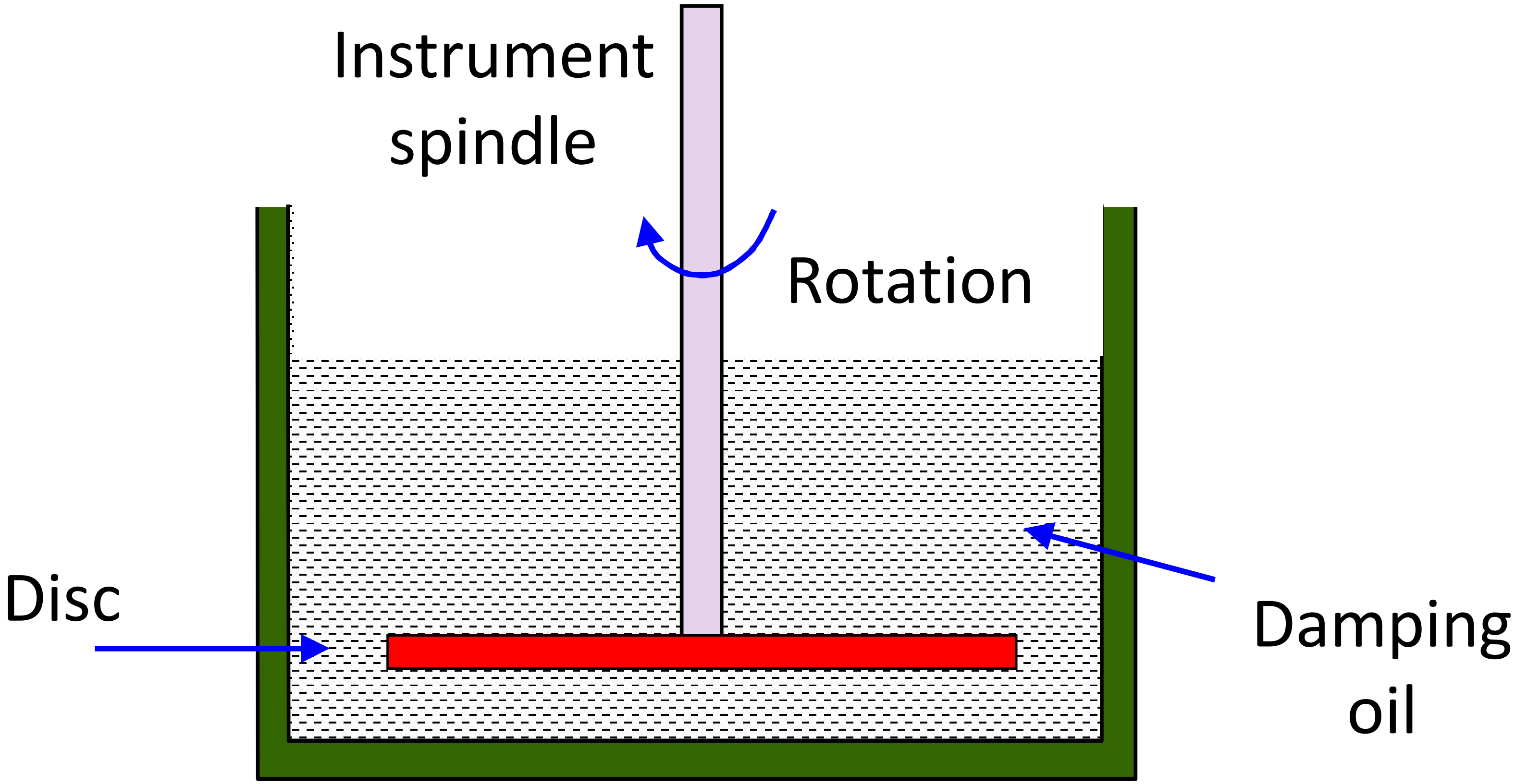
Oil damping requires that the instrument is kept in the ***vertical position*** and it is unsuitable for portable instruments

Dynamic response

Damping Torque

Mechanical damping

Liquid damping



Dynamic response

Damping Torque

Mechanical damping

Solid friction damping

Solid friction "pivot friction" is a normal friction due to the mechanical movement

The friction torque, which is not a function of angular velocity, is low enough to be neglected

The mechanical damping is given by:

$$\boxed{T_{dm} = D_m \frac{d\theta}{dt}}$$

Dynamic response

Damping Torque

Electromagnetic “eddy current” damping

This is the most efficient damping method

A thin disc of a conducting, but non-magnetic, material like copper can be mounted as a frame to the moving system and the pointer of the instrument

When the disc rotates, its edges cut the magnetic flux produced by the poles of a permanent magnet

The rotation of the coil inside the magnetic field sets up eddy currents circulating in the conductive metal frame

Dynamic response

Damping Torque

Electromagnetic “eddy current” damping

The flow of the eddy currents produces a damping force in an opposite direction to that produced them according to Lenz's Law

This causes a retarding torque in opposite direction to the motion of the coil and the pointer

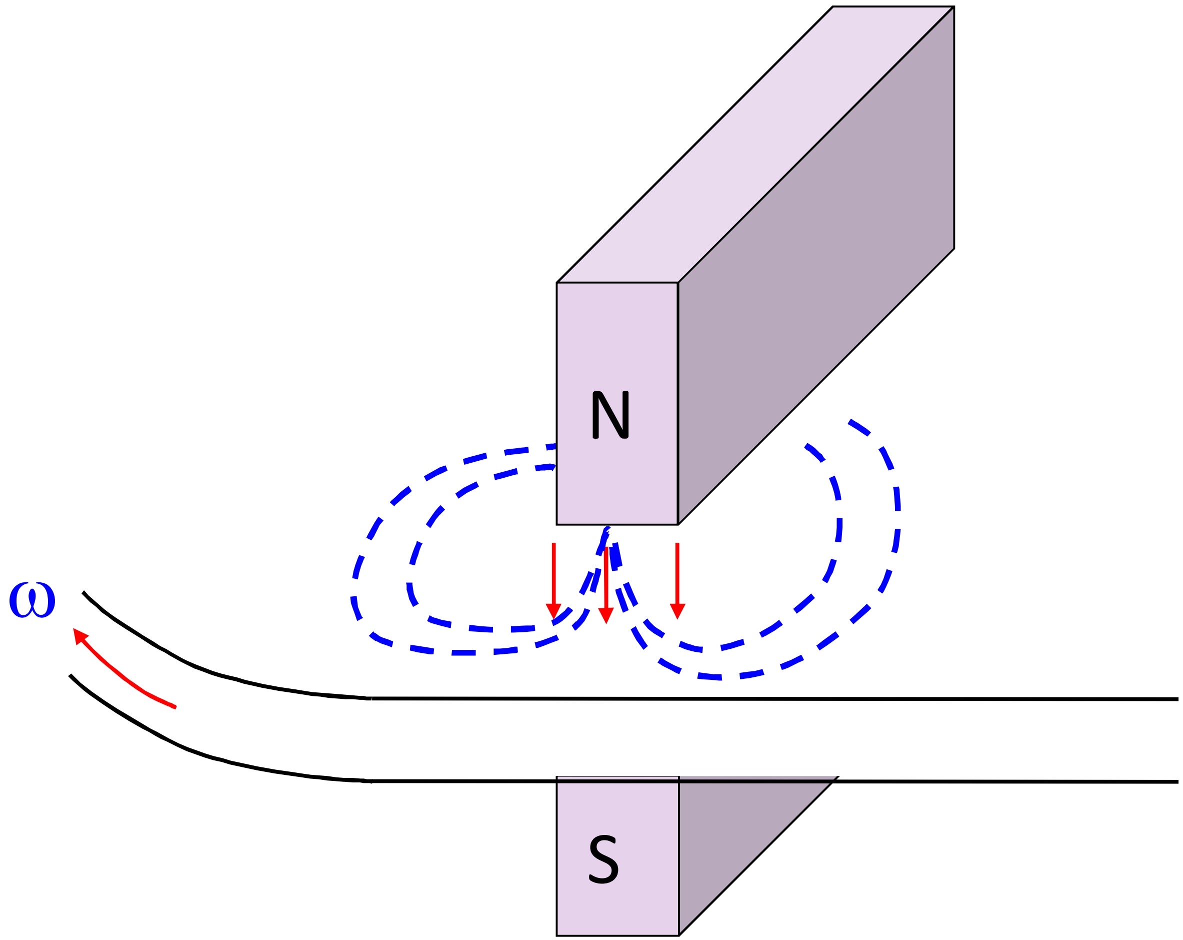
The coil can be wound on a thin light aluminium former in which eddy currents are produced when the coil rotates

This type of torque is called “electromagnetic damping torque”

Dynamic response

Damping Torque

Electromagnetic “eddy current” damping



Dynamic response

Damping Torque

Electromagnetic “eddy current” damping

The eddy-current-damping torque is given as:

$$\boxed{T_{de} = De\frac{d\theta}{dt}}$$

The total damping torque is given by

$$\boxed{T_d = T_{dm} + T_{de}}$$

Dynamic response

Damping Torque

Electromagnetic "eddy current" damping

The equivalent damping constant is given by:

$$\boxed{D = D_m + D_e}$$

The total damping torque is given by:

$$\boxed{T_d = D\frac{d\theta}{dt}}$$

Dynamic response

Solution of the dynamic equation

The equation of motion

$$\underbrace{T}_{\text{Deflecting torque}} = \underbrace{\overset{\text{Inertia}}{J}\,\frac{d^2\theta}{dt^2}}_{\text{Inertia torque}} + \underbrace{\overset{\text{Damping constant}}{D}\,\frac{d\theta}{dt}}_{\text{Damping torque}} + \underbrace{\overset{\text{Control-torque constant}}{C}\,\theta}_{\text{Control torque}}$$

Dynamic response

Solution of the dynamic equation

The equation of motion

$$\boxed{\mathrm{K}f(\mathrm{I}) = J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta}$$

Solving this 2nd - order differential equation gives the relationship between deflection angle “θ” and time “t”

The behaviour of the instrument depends on the solution of the equation and the relation between the three constants defines the type of performance

Dynamic response

Solution of the dynamic equation

The solution of the equation can take three forms

The first case is the **over-damped** performance

The pointer moves slowly to its final value without oscillations

The absence of the oscillations represents an advantage for this case but the slow performance represents a main disadvantage

The condition of this situation is given as:

$$\boxed{D > \sqrt{4CJ}}$$

Dynamic response

Solution of the dynamic equation

The solution of the equation can take three forms

The second case is the **under-damped**

The pointer moves very fast but with high oscillations

The performance is characterized by the damped behaviour and the oscillations decay with time

This type of performance is not favourable since the pointer will take a long time to reach steady state

$$\boxed{D < \sqrt{4CJ}}$$

Dynamic response

Solution of the dynamic equation

The solution of the equation can take three forms

The third case is the **critical-damped** performance

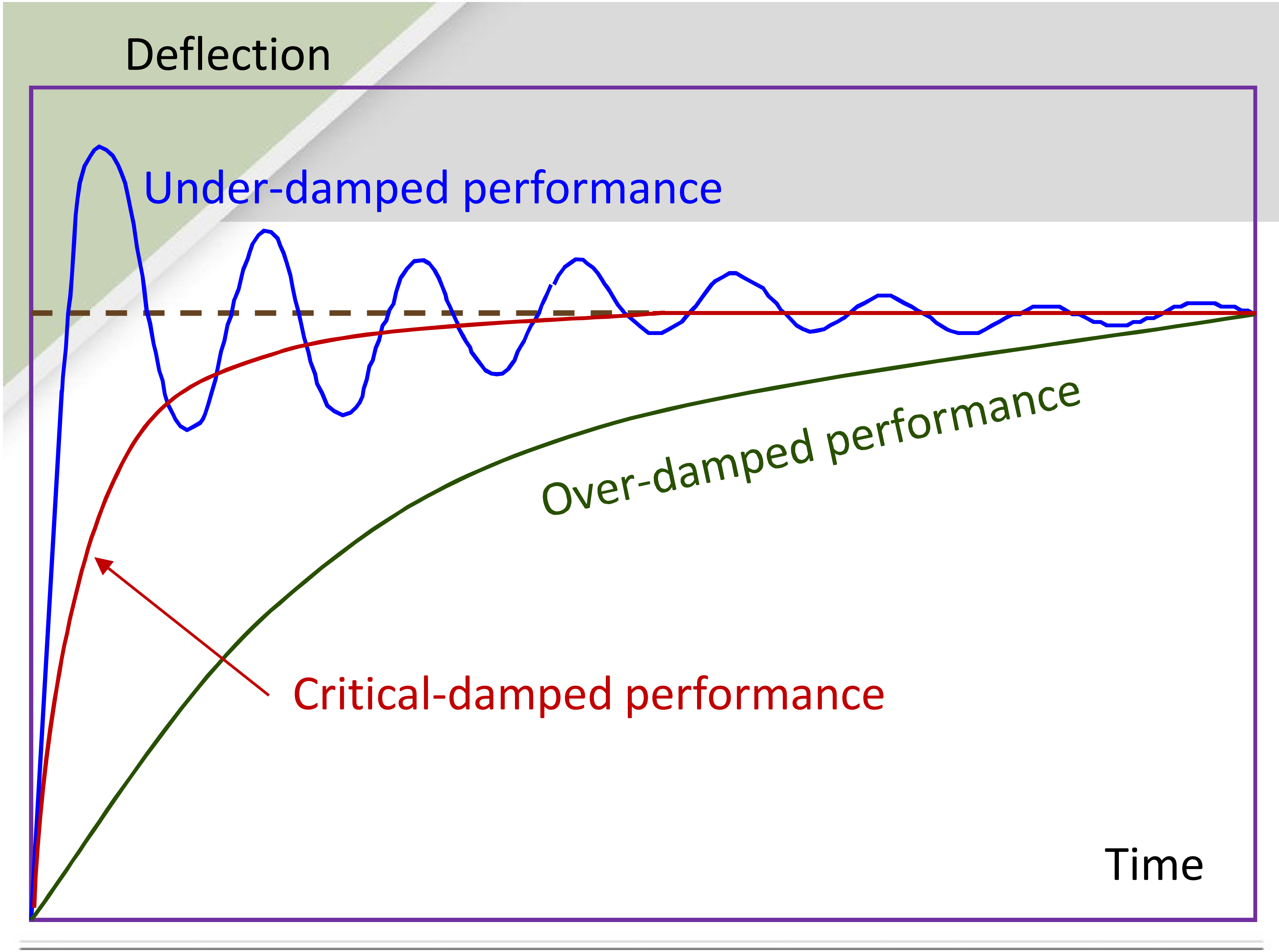
The pointer moves faster than the over-damped case and slower than the under-damped case

The movement takes place without any oscillations

If the pointer moves a little bit faster than this situation, oscillations start to appear

Therefore, this case is termed “critical-damped”

$$\boxed{D = \sqrt{4CJ}}$$


Deflection
Under-damped performance
Over-damped performance
Critical-damped performance
Time